Characteristics of a Radiating Layer Near the

Boundary of a Contaminated Plasma

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Abstract

The properties of a radiating boundary layer near the edge of a hot plasma containing impurities are considered. A discontinuous sheath model is developed which includes heat conduction, radiation and additional heating (or losses). Simple analytic scaling formulas are obtained for the equilibrium, and means of external control are suggested. Stability criterions are derived for various sheath structures, and growth rates and eigenfunctions are given for an unstable case. The results may help to understand the dynamics of a radiating boundary layer and may be used as a rough guideline for numerical plasma simulations and even for experiments.

1. Introduction

An important question in large fusion experiments or a reactor is how to get rid of the plasma energy losses at the wall without damaging the wall and/or producing such contamination that the plasma as a whole cools down by radiation /1, 2, 3/. It was argued that energy transport to the wall by neutral or charged particles at relevant power densities necessarily leads to impurity production until nearly all the outgoing energy is radiated away by the increased impurity radiation /4/.

For the ignition experiment ZEPHYR proposed at Garching such a self-shielding would be vital, since, because of the small dimensions, the wall loading is indeed extremely high. Therefore, plasma simulations with the one-dimensional computer code BALDUR /5/ developed at Princeton were made in order to study the consequences of various wall inter-action models and their compatibility with an ignited plasma state.

The desired behaviour was indeed observed, for instance in case of an idealized radiofre-quency heating (prescribed heat deposition profile with some relevance for ion cyclotron heating):

For a certain amount of impurity, e.g. iron, a cold plasma boundary layer developed which was separated from the hot plasma by a narrow radiation zone where nearly 100 per cent of the outgoing energy flux was radiated away. This sheath structure could be maintained for several hundred milliseconds in the case of a prescribed iron influx. An even more optimistic result was obtained when the iron influx was coupled to the charge exchange outflux via sputtering. In this case a self-regulating mechanism was observed in the sense that sputtering (and therefore the iron influx) stops when the radiation layer has developed and the high-energy charge exchange neutral flux to the wall is suppressed. On a long time scale one may expect an equilibrium structure of the sheath such that the diffusion of iron to the wall is equal to the influx. In sputtering runs the radiation sheath could be frequently maintained during the heating cycle (simulating RF heating; no compression) and the sub-sequent burn phase, i.e. for several seconds.

Figure 1a shows typical profiles calculated with BALDUR (iron sputtering) 0.6 seconds after the end of a 1-second RF heating pulse (11 MW in a 20 cm thick toroidal cylindrical shell; R_o = 135 cm, a = 50 cm, B = 9.15 tesla). There is a cold plasma layer ($T_e \approx$ several e V), roughly identical with the density gradient region, separated from the hot plasma by a narrow

radiation sheath as seen in Fig. 1b, where the radially integrated losses and heating rates are shown. The total α - heating is higher than the sum of all losses, i.e. the plasma is ignited. About 94 % of the energy loss at the wall is by radiation, about two-thirds originating from a narrow sheath separating the cold plasma layer from the hot central plasma. There is still some loss by charge exchange neutrals caused by the increasing α - input, which, in turn, increases the iron content and readjusts the radiation level. The amount of iron impurity, typically below 0.05 per cent, is still compatible with ignition, which means the radiation from the hot central plasma is low enough.

The sheath width in the above BALDUR run is still influenced by the finite grid size since the mean free path of neutrals in the boundary layer is still comparable with the grid distance. Refining the grid in the boundary layer simply leads to a steeper density gradient (gas puffing plus recycling) and a smaller sheath, while the sheath structure is maintained. This is demonstrated in Fig. 2 , which shows profiles during a 15 MW RF heating pulse, which again leads to ignition about half a second later. Further details of these BALDUR simulations and the sheath dynamics during a heating cycle will be given elsewhere. Of course, additional impurities and various physical mechanisms not included in BALDUR (e.g. deviations from local coronal equilibrium) might change this picture and iron may not be the optimum impurity, i.e. first wall material. Optimization will certainly require further experiments and theoretical progress.

In any case such a radiation-cold plasma layer would obviously ease the wall loading problem dramatically. For ZEPHYR one expects an average wall loading of around 50 W/cm^2 in the case of uniformly distributed radiation, while heat conduction to the wall might well reach $\geqslant 1$ kW/cm^2, depending on the effective area where the plasma touches the wall. The first value is quite reasonable while the second is nearly prohibitive.

For a qualitative understanding of the dynamics and the parameter scaling of such a cold plasma - radiation double layer it is useful to look at a simple analytic model like that described below.

II. Boundary Layer Equilibrium

1. The Basic Sheath Model

We assume a semi-inifinite plasma slab with spatially constant electron, hydrogen and impurity density (n_e, n_H, n_i) but arbitrary temperature dependence:

$$n_e, n_H, n_i = const; \quad T = T(x).$$

The spatial coordinate x is zero at the boundary and positive in the plasma. Energy transport in the x direction occurs by heat conduction and radiation only. Charge exchange neutrals are not considered. According to the BALDUR result, this model should include the most important mechanisms relevant for the existence of the radiation layer and its stability at least on a time scale compared with the characteristic time of change of the density profiles. The effect of a density gradient will be considered later on. The model obviously does not include the immediate neughbourhood of the limiter or wall, which is much more complicated. This transition layer can at best be taken into account by properly chosen boundary conditions (the same is true of BALDUR).

A further important simplification of our model is obtained by choosing an idealized radiation characteristic or cooling rate S(T) caused by the impurity

$$S(T) = S_o = const \text{ for } T_1 < T < T_2$$
$$S = 0, \text{ elsewhere.}$$

(see Fig. 3)

The justification for this may be drawn from the cooling rates given in the literature /6/, at least in the case of low and intermediate atomic weight. In addition, a rapid impurity diffusion, which might appreciably modify the local coronal equilibrium assumption /7/, could be qualitatively taken into accound by using a broader radiation characteristic, i.e. by increasing T_2 and appropriately decreasing T_1. The same is possible if several impurities radiate simultaneously at a comparable level in overlapping or neighbouring temperature regions. Because of the constant density, the radiated power density P is directly proportional to the normalized cooling rate S:

$$P \propto n_e n_i \cdot S \propto S$$

The desired equilibria are specified as follows: A given heat flux Q_{III} from the hot plasma (region III; $T > T_2$) enters a radiation layer (region II; $T_1 < T < T_2$), where an appreciable fraction is radiated away while the rest is transported to the wall through a colder plasma layer (region I; $T < T_1$). For simplicity, the wall temperature is set to zero, but obviously it could have any other value below T_1. (Notation: Indices 1, 2 refer to the boundaries; I, II, III refer to the relevant regions).

From the heat conduction $Q = \chi \cdot dT/dx$ (Q positive in the direction of the wall; χ = heat conduction coefficient) and energy conservation $dQ/dx = P$ we get $\chi \, d^2T/dx^2 = P(T)$, which is easily integrated in each region because of the very special assumption for $P(T)$. Prescribing P_o, T_1, T_2, Q_{II}, χ_I, χ_{II} we immediately get the width a of region I, the width b of region II and the remaining heat flow to the wall, Q_I:

$$a = \frac{\chi_I T_1}{Q_I} = \chi_I T_1 \Big/ \sqrt{Q_{III} - 2\chi_{II} P (T_2 - T_1)}$$

$$b = \frac{Q_{III}}{P} \left(1 - \sqrt{1 - 2\chi_{II}(T_2 - T_1)P/Q_{III}^2}\right)$$

$$Q_I = \sqrt{Q_{III}^2 - 2\chi_{II} P (T_2 - T_1)}$$

A typical solution indicating a reasonable similarity with the BALDUR result is shown in Fig. 4.

Except for the obvious scalings of a, b, Q_I with T_1, T_2, P_o, Q_{III} one should notice the consequences of changing χ_I and/or χ_{II}. For instance, the width of the radiation layer b and therefore the total radiated power, $P_{tot} = P_o \cdot b$, can be increased by increasing χ_{II} until $P_{tot} = Q_{II}$, $b = b_{max} = Q_{II}/P_o$ and $Q_I = 0$ (fixed impurity characteristic, i.e. T_1, T_2, P_o). This also means that by increasing χ_{II} one can decrease the impurity content necessary to radiate away Q_{III} or a large fraction of it.

In a similar way the width of the cold plasma layer can be increased by increasing χ_I, but in contrast to the radiation zone b it does not reach an asymptotic value. Note also that the

width a is directly proportional to T_1 (which is not very well defined experimentally), and goes to infinity when Q_1 approaches zero. (Near this limit it is rather sensitive to inaccuracies of the model and the numerical value should not be taken too seriously).

The sensitivity of the radiation – cold plasma layer to changes in the thermal conduction coefficient could be used to influence this layer externally (e.g. by ergodization of field lines) and could even be used to explain certain features of the disruption of contaminated plasmas /8/.

2. Extensions of the Basic Sheath Equilibrium
a) Additional heating in the sheath region

It is now assumed that there is piecewise homogeneous additional heating throughout the sheath (e.g. from a penetrating neutral beam) in competition with the radiation loss, which occurs only in region II. With a heating power density H_I and H_{II} we immediately get

$$a = \sqrt{\frac{Q_1^2}{H_I^2} + \frac{2\chi_I T_1}{H_I}} - \frac{Q_1}{H_I}$$

$$b = \frac{Q_2}{P - H_{II}} - \sqrt{\left(\frac{Q_2}{P - H_{II}}\right)^2 - \frac{2\chi_{II}(T_2 - T_1)}{P - H_{II}}} \qquad \left(P > H_{II}\right)$$

$$Q_1 = Q_2 - \sqrt{Q_2^2 - 2\chi_{II}(T_2 - T_1)(P - H_{II})}$$

It should be noticed that for $H_I = 0$ or $(P - H_{II}) = 0$ we get $a = \chi_I T_1 / Q_1$ and $b = \chi_{II}(T_2 - T_1)/Q_2$ as expected.

The result for b differs from the previous one only in $(P - H_{II})$ instead of P.

More interesting is the influence on a: for $Q_1 \to 0$ we no longer get $a \to \infty$, but

$$a_{max} = \sqrt{\frac{2\chi_I \cdot T_1}{H_I}}$$

This means that the width of the cold plasma layer, a, can be externally controlled by

additional heating in this region, especially in the important case where Q_1 is small (see also / 1 /). A jump in H may also occur somewhere away from the radiation boundaries, depending on the heating mechanism. An example is sketched in Fig. 5.

b) Density gradients

We take the basis model but allow for a gradient in the densities; χ is kept constant. Because of our definition of the heat conduction coefficient, $Q = \chi \, dT/dx$, $\chi =$ const is equivalent to the well-known $\kappa \propto n^{-1}$ scaling in the usual tokamak definition $(Q = \kappa \, n \, dT/dx)$: $\chi = n \cdot \kappa$

The radiated power is now a function of T and x

$$P(T,x) \propto n_e(x) \cdot n_j(x) \cdot S'(T) \propto \begin{cases} n_e(x) \cdot n_j(x), & T_1 < T < T_2 \\ 0, & \text{elsewhere} \end{cases}$$

The equilibrium is again obtained simply by integrating $\chi \, d^2T/dx^2 = P(T,x)$ for a prescribed $n_e(x) \cdot n_i(x)$. Though these equilibria are quantitatively different from the previous ones, they show essentially the same scaling with χ, T_1, T_2 etc. as for constant density. In fact, it is the stability of the radiating layer which is sensitive to density gradients, as will be shown later for a special example (Fig. 6).

c) Multistep radiation characteristics

A more realistic approximation for the radiation characteristics is obtained by using a multistep function. For the case with constant radiation loss in regions I and III in addition to the major radiation loss in II we shall investigate the stability. The equilibrium is again not much different from those above (Fig. 7).

d) Spatially fixed jump in heat conduction coefficient

A case of special interest is that where χ jumps somewhere in the sheath. If this jump is caused by a region of ergodic field lines or a separatrix, it will be spatially fixed and may therefore help to stabilize the radiation zone.

e) Temperature dependent χ, etc.

There are further possible extensions of the sheath model, such as $\chi = \chi(T)$ etc. Some of these require a modified analytical treatment, e.g. a differential equation in the stability analysis other than those chosen above (which lead to a homogeneous heat conduction equation; see below).

It is not our intention to give a complete summary of all possible equilibria, but to demonstrate the main features of a radiating boundary layer, for as simple examples as possible and to get simple scaling laws and stability criteria.

III. Stability of the Radiating Layer

1. Stability of the Basic Sheath Structure

Because of the simple model it is possible to make a stability analysis for the radiation layer. The equation for T (x, t) is

$$\frac{\partial T}{\partial x} = C\left(\chi \frac{\partial^2 T}{\partial x^2} - P(T)\right) \; ; \quad \chi_{\parallel} = \chi_I = \chi$$

where C is inversely proportional to the heat capacity.

Assuming a perturbed temperature

$$T(x,t) = T_0(x) + \tau(x,t) \quad , \quad \tau \ll T_0$$

we get the linearized equation

$$\frac{\partial \tau}{\partial t} = C\chi \frac{\partial^2 \tau}{\partial x^2}$$

within each region, i.e. we get the homogeneous heat conduction equation with the solution

$$\tau = \exp(\gamma t)\left[k_1 \exp\left(+\sqrt{\gamma/(C\chi)}\cdot x\right) + k_2 \exp\left(-\sqrt{\gamma/(C\chi)}\cdot x\right)\right]$$

The main question is whether there are unstable solution ($\gamma > 0$) and, if so, what are the growth rates. We are not so much interested in the details of stable (spatially periodic) solutions ($\gamma < 0$).

A special feature of the discontinuous model is that a perturbed temperature is directly related to a shift of the boundaries between the sheaths (coordinates $x_1 = a$; $x_2 = a + b$). Assuming shift $\tilde{x}_1$ and $\tilde{x}_2$ respectively and requiring that

$$T\left(x_{\substack{10\\20}} + \tilde{x}_{\substack{1\\2}}\right) = T_0\left(x_{\substack{10\\20}}\right) = T_{\substack{1\\2}}$$

(the index 0 refers to equilibrium quantities), we find that

$$\frac{\partial T_0}{\partial x} \cdot \tilde{x}_{\substack{1\\2}} = -\tau\left(x_{\substack{10\\20}}\right)$$

$\tilde{x}_1, \tilde{x}_2$ do not enter directly into the analysis, but are used to determine the jump condition for the heat flux across the boundaries at x_1 and x_2.

The heat flux lost at the boundary is

$$\delta Q = \int_0^{\tilde{x}_{\substack{1\\2}}} \left(\frac{\partial Q}{\partial x} - P\right) d\tilde{x} = \begin{cases} P\tilde{x}_1 = -\tau(x_{10})\mathcal{X}\,P/Q_I \quad , \quad x = x_{10} \\[2mm] -P\tilde{x}_2 = \tau(x_{20})\mathcal{X}\,P/Q_{III} \quad , \quad x = x_{20} \end{cases}$$

It is the δQ which represents the driving (δQ_2) and stabilizing (δQ_1) terms or, in other words, increasing (decreasing) radiation with increasing temperature is stabilizing (destabilizing).

The boundary conditions are now

$x = 0:$ $\tau_I(0) = 0$ $\left(\text{or } \partial\tau_I/\partial x\,(0) = 0, \text{ or mixed}\right)$

$x = x_{10}:$ $\tau_I(x_{10}) = \tau_{II}(x_{10})$

$$\frac{\partial \tau_{II}}{\partial x}(x_{10}) - \frac{\partial \tau_I}{\partial x}(x_{10}) = +\frac{P}{Q_I}\,\tau_{I,II}(x_{10})$$

$x = x_{20}:$ $\tau_{II}(x_{20}) = \tau_{III}(x_{20})$

$$\frac{\partial \tau_{III}}{\partial x}(x_{20}) - \frac{\partial \tau_{II}}{\partial x}(x_{20}) = -\frac{P}{Q_{III}}\,\tau_{II,III}(x_{20})$$

$$X = \infty : \quad \tau_{III}(\infty) = 0 \qquad \left(\frac{\partial \tau_{III}}{\partial x}(\infty) = 0 \right)$$

(or some mixed condition at finite x_{30}).

Accordingly, we choose an ansatz for the eigenfunction of the form

$$\tau_{I \atop II} = \exp(\gamma t)\left[k_1^{I \atop II} \exp(\varepsilon x) + k_2^{I \atop II} \exp(-\varepsilon x) \right],$$

$$\tau_{III} = \exp(\gamma t)\, k_2^{III} \exp(-\varepsilon x),$$

$$\varepsilon = \sqrt{\gamma/(C'\chi)}, \qquad \gamma = \varepsilon^2 C' \chi$$

Inserting this in the boundary conditions, we get a set of linear equations for the coefficients k. Nontrivial solutions are obtained if the determinant D is zero, yielding the dispersion relation

$$D = \frac{P}{Q_{III}} e^{-\varepsilon b} \left[\varepsilon \cosh(\varepsilon a) - \left(\varepsilon - \frac{P}{Q_I}\right) \sinh(\varepsilon a) \right] +$$

$$+ \left(2\varepsilon - \frac{P}{Q_{III}}\right) e^{+\varepsilon b} \left[\varepsilon \cosh(\varepsilon a) + \left(\varepsilon + \frac{P}{Q_I}\right) \sinh(\varepsilon a) \right] \doteq 0$$

This is now an equation for ε (and therefore γ), which cannot be solved analytically. One can, however, draw some conclusions from the behaviour of $D(\varepsilon)$ for $\varepsilon \to 0$ and $\varepsilon \to \infty$:

$$D(\varepsilon = 0) \equiv 0 \qquad\qquad D'(0) = dD/d\varepsilon \,(0) \equiv 0$$

$$D(\varepsilon \to \infty) \to \infty$$

$$D''(0) = d^2D/d\varepsilon^2 \,(\varepsilon=0) \propto 1 + a\frac{P}{Q_I} - (a+b)\frac{P}{Q_{III}} - ab\frac{P^2}{Q_I Q_{III}}$$

$$\propto Q_I^2 \,, \text{ since } P = (Q_{III} - Q_I)/b \,.$$

Since $Q_1 \geqq 0$ (monotonically increasing temperature) was assumed, one finds

$$D''(0) \geqq 0$$

a condition which is necessary, but not sufficient for stability ($D''(0) < 0$ would be sufficient but not necessary for <u>instability</u>! See Fig. 8).

In a numerical study of D (ε) for a variety of such basic sheath structures, we never found an unstable one, i.e. the criterion $D''(0) > 0$ may be an exact stability criterion.

In order to investigate the influence of the boundary condition at the wall, the flux was fixed instead of the temperature, i.e. $\partial \tau / \partial x$ instead of τ. The result, however, is essentially the same, namely that all reasonable sheaths examined so far are stable.

2. Stability of More Complex Sheath Structures

As mentioned earlier, the extended equilibria described above are chosen such that they can be treated with the same stability formalism as the basic sheath. In fact, they all lead to the homogeneous heat conduction equation after linearization and only minor modifications of the formalism are required, which are not described in detail. Usually we give only the modified stability criterion, but in one case growth rates and eigenfunctions are also presented.

a) Additional heating in the sheath region

If there is a constant additional heating power H (or a nonradiative loss) throughout the sheath region, then the equilibrium is changed but the stability is essentially as for the basic sheath, i.e. the sheath is stable.

If there is a jump of the heating power somewhere fixed in space (in reality this would mean a strong gradient), we still find stability.

A major change occurs only if the jump in the heating power (or loss) is not spatially fixed but temperature dependent and undergoes a shift when the temperature changes. This case may well be of practical interest since many heating or loss mechanisms depend on the local temperature and there is an extreme temperature gradient over the sheath. The additional heating or loss may then be treated like radiation losses taking the appropriate sign.

A special case where the jump coincides with one of the boundaries of the radiation sheath is easily treated by combining temperature dependent heating and loss to obtain an artificial "radiation" characteristic. This case is considered quantitatively in (c). Here, we anticipate the qualitative result: If the total jump is decreased at the low-temperature edge and/or increased at the high-temperature edge of the radiation layer, then it is destabilizing and vice versa. The heating power should thus decrease towards the hot plasma interior to enhance stability (or losses should increase).

Physically relevant loss mechanisms of this type are ohmic heating or ionization of neutral gas (gas puffing and recycling) and of impurities in the boundary layer, where the cross-section is a steep function of temperature.

In principle, we are led to the simple conclusion that the combined energy deposition characteristic is locally stabilizing if losses increase (heating decreases) with increasing temperature and vice versa. It is the purpose of the present analysis to calculate the net stability effect in a spatially extended system consisting of locally favourable and unfavourable regions by solving the corresponding eigenvalue problem.

b) Spatially varying density

If the radiating layer lies in a density gradient region, the an average shift to higher densities increases the local radiation power ($P \propto n_e \cdot n_j$). It depends on the change of the layer thickness and the relative sign of the density and temperature gradient whether the layer becomes unstable or not.

In order to get a rough estimate of the stability in this case, we take equilibria of the type described in Sec. II b, but assume a rather simple form for the x-dependence of $P(T,x)$:

$$P(x,t) \propto S(T)\left[P_1 + (P_2 - P_1)(x-a)/b\right] , \quad a \le x \le a+b ,$$

$$(P_1 + P_2)/2 = P = (Q_{III} - Q_I)/b$$

This means that P increases linearly with x from P_1 to P_2 across region II. This type of

equilibrium is sketched in Fig. 6.

It is convenient to describe the equilibrium by Q_I, Q_{III}, a, b and $V = P_1/P_2$. These quantities could be expressed in terms of a more physical set of given quantities, e.g. Q_{III}, X, T_1, T_2, S_o etc. The corresponding relations, however, are not required in the stability analysis and are therefore not explicitly calculated.

Constant heat capacity, C = const. is assumed in order to retain the type of differential equation. This is valid only if n_e = const, $n_i = n_i(x)$ and $n_i \ll n_e$. However, the marginal stability limit, i.e. the stability criterion, nevertheless also applies to the general case, $n_e = n_e(x)$, $n_i = n_i(x)$.

The stability analysis proceeds as before except that now

$$\delta Q = \begin{cases} P_1 \widetilde{X}_1 = - \tau(x_{10}) \, X \, P_1/Q_I \, , & X = X_{10} \\ P_2 \widetilde{X}_2 = + \tau(x_{20}) \, X \, P_2/Q_{III} \, , & X = X_{20} \end{cases}$$

and in the determinant D and in the stability criterion one has to replace

$$P/Q_I \;\to\; P_1/Q_I \, , \qquad P/Q_{III} \;\to\; P_2/Q_{III}$$

With the boundary condition $\tau(0) = 0$, i.e. fixed temperature at the boundary, we get as a sufficient (not necessary) criterion for instability

$$1 + a \, \frac{P_1}{Q_I} - \frac{P_2}{Q_{III}} (a+b) - \frac{P_1 P_2}{Q_I Q_{III}} \, ab < 0 \, ,$$

$$P_1 = 2(Q_{III} - Q_I)/(b(1+V)) \, , \qquad P_2 = V P_1$$

In contrast to the basic sheath ($P_1 = P_2 = P$), we now also find unstable solutions if P_2/P_1 is above a certain level.

A simpler criterion is obtained in the weak gradient approximation:

$$P_1 = P_0 (1 - \eta)$$
$$P_2 = P_0 (1 + \eta) \quad , \quad \eta = \frac{P_2 - P_1}{P_2 + P_1} \quad , \quad |\eta| \ll 1 \ .$$

The criterion for instability becomes

$$\eta > \left[\frac{a}{b} \left(\frac{Q_{III}^2}{Q_I^2} - 1 \right) + \frac{Q_{III}}{Q_I} - 1 \right]^{-1}$$

and if in addition

$$\frac{a}{b} \gg \frac{Q_I}{Q_{III}} \quad , \quad Q_{III} \gg Q_I$$

then

$$\eta > \frac{b}{a} \frac{Q_I^2}{Q_{III}^2}$$

for instability.

Indeed, for the most interesting equilibria ($Q_I \ll Q_{III}$, $a \approx b$) we get instability even for a very weak density gradient if density and temperature increase in the same direction.

For the physically less meaningful boundary condition $\partial \tilde{\tau} / \partial x = 0$ at the wall (fixed heat flux), the criterion is even more stringent:

$$\eta > 0 \ , \qquad \text{unstable,}$$

i.e. any $P_2 > P_1$ causes instability, while $P_2 \le P_1$ (different sign of temperature and $n_e \cdot n_i$ gradient) yields improved stability (necessary criterion).

For a typical equilibrium the unstable eigenvalues ε (or growth rates $\gamma = \varepsilon^2 c \chi$) are calculated numerically from $D(\varepsilon) \equiv 0$. The result is shown in Fig. 9 a) and b), where $\varepsilon \cdot b$ is plotted versus $V = P_2/P_1$ and $\eta = (V-1)/(V+1)$, respectively, for both types of boundary conditions. The analytic criterion is also given and turns out to coincide with the numerical marginal points. The criterion thus seems to be necessary and sufficient.

The absolute value of the growth rate can be interpreted in terms of a heat conduction time $t_{H.c.}$ over a sheath width Δ as follows:

From the heat conduction equation and a model sheath, $T = T_o \cdot x^2 / \Delta^2$ we get

$$\frac{1}{t_{H.C.}} = \frac{1}{T} \frac{\partial T}{\partial t} = \frac{CX}{T} \frac{\partial^2 T}{\partial x^2} = 2CX/\Delta^2$$

and

$$\gamma = CX\varepsilon^2 = \frac{1}{2 t_{H.C.}} (\varepsilon \Delta)^2$$

The maximum εb value found in the example for $P_2/P_1 \gg 1$, $\varepsilon b \approx 0.8$ therefore corresponds to an e-folding time τ_e of the instability which is roughly three times the typical heat conduction time over the region II:

$$\tau_e = \frac{1}{\gamma} = 2 t_{H.C.} / (\varepsilon b)^2 \approx 3 t_{H.C.}$$

For an arbitrarily chosen case, $V = 3$ and $\varepsilon b \approx 0.4$, the corresponding eigenfunction is shown in Fig. 10. It is recalled that a positive τ at a sheath boundary corresponds to an outward shift (smaller x) of that boundary (and vice versa), the magnitude being dependent on τ and the local equilibrium temperature gradient (Sec. III, 1). In the above example, the shift $\tilde{x}_1$ is slightly larger than $\tilde{x}_2$. Thus, a positive temperature disturbance causes an outward shift of the sheath and the sheath width increases in parallel. But the total radiated power decreases since $|\delta Q_1| < |\delta Q_2|$ allowing a further increase in temperature and driving the sheath unstable.

c) Multistep radiation characteristics
(including density gradient and additional heating)

The stability criterion for equilibria with multistep radiation characteristics (Fig. 7 and Sec. II, c) is simply obtained from that in Sec. III a by replacing

$$\frac{P}{Q_I} \rightarrow \left(P_{II}(a) - P_I(a) \right) / Q_I(a) = \Delta P_1 / Q_1$$

$$\frac{P}{Q_{III}} \rightarrow \left(P_{II}(b) - P_{III}(b) \right) / Q_{III}(b) = \Delta P_2 / Q_2$$

since the only significant change is again for the value of δQ at the free boundaries

(Q_I and Q_{III} are no longer constant in the respective regions).

In order to get a simple and explicit form of the criterion we assume that the radiation power in regions I and III is small compared with that in region II (ν_1, $\nu_2 \ll 1$, see below). In addition, a small density gradient ($|\eta| \ll 1$) may exist.

$$P_I(a) = \nu_1 P_0 \qquad\qquad P_{II}(a) = P_0(1-\eta)$$

$$P_{III}(b) = \nu_2 P_0 \qquad\qquad P_{II}(b) = P_0(1+\eta)$$

Neglecting $\nu \cdot \eta$ terms, the criterion for instability is

$$0 < 1 + \frac{a(Q_2-Q_1)}{b \cdot Q_1}(1-\eta-\nu_1) - \frac{Q_2-Q_1}{Q_2}\left(\frac{a+b}{b}\right)(1+\eta-\nu_2) - \frac{a}{b}\frac{(Q_2-Q_1)^2}{Q_2 Q_1}(1-\nu_2-\nu_1)$$

In the most interesting limit, $Q_1/Q_2 \lesssim \nu \ll 1$ and $a \approx b$, the criterion reduces to the simple form

$$0 \leqslant \nu_2 - \eta \qquad or \qquad \eta \leqslant \nu_2$$

for instability. This means that a high-temperature tail in the radiation characteristic reaching far into the hot plasma ($\nu_2 > 0$) has a stabilizing influence on the radiating sheath and therefore a higher density gradient can be tolerated. The small radiation in region I ($\nu_1 > 0$) has much less influence on stability.

The above criteria also apply quantitatively to the case of radiation losses plus temperature dependent heating power if the latter has a jump at the same temperature as the radiation. Some practical consequences have already been discussed in Sec. III, 2a).

d) Spatially fixed jump of χ (including a density gradient)

A single, spatially fixed jump of χ at an arbitrary x = x* would complicate the model by introducing a third internal boundary. This difficulty is circumvented by choosing the jump in χ to be close to one of the boundaries of the radiating layer. It can then be

accounted for by properly reformulating the boundary conditions at that layer.

α) Jump of χ in region II close to x_1 (Fig. 11a)

The position of the jump is denoted by x^* and it is assumed that $x^* - x_1 \ll b$ and positive; x^* separates region II in IIa and IIb (Fig. 11a). The variation of the perturbed temperature τ over IIa is negligible ($\varepsilon(x^* - x) \ll 1$) and therefore τ is continuous as before. The flux jumps at x_1 by δQ_1, but is continuous at x^*, which means

$$\chi_{IIa} \frac{\partial \tau_{IIa}}{\partial x} = \chi_{IIb} \frac{\partial \tau_{IIb}}{\partial x}$$

$$\chi_{IIa} = \chi_I \; , \quad \chi_{IIb} = \chi_{III}$$

The modified boundary condition at $x_1 \approx x$, separating region I and IIb $\hat{\approx}$ II, is then

$$\frac{\chi_{II}}{\chi_I} \frac{\partial \tau_{II}}{\partial x} - \frac{\partial \tau_I}{\partial x} = \frac{P}{Q_I} \cdot \tau_{I,II}$$

All other boundary conditions are as before (Sec. III,1). Inserting the ansatz for (now $\varepsilon_{I,II} = \sqrt{\gamma/(C \cdot \chi_{I,II})}$) and calculating the determinant of the system of linear equations, we arrive at a rather complicated criterion for instability which for $Q_1 \ll Q_2$, $a \approx b$ reduces to

$$\eta \gtrsim \frac{Q_1}{Q_2} \left(1 - \frac{\chi_{II}}{\chi_I}\right)$$

Obviously, $\chi_{II} < \chi_I$ is stabilizing and allows for positive $\eta \lesssim \frac{Q_1}{Q_2} \ll 1$, but the effect is rather small.

β) Jump of χ in region II close to x_2 (Fig. 11b)

Region IIb now shrinks to a small width ($x_2 - x^* \ll b$ and positive) and the modified flux condition at x_2 becomes

$$\frac{\partial \tau_{III}}{\partial x} - \frac{\chi_{II}}{\chi_{III}} \cdot \frac{\partial \tau_{II}}{\partial x} = -\frac{P}{Q_{III}} \tau_{III,II} \; ; \qquad \begin{array}{l} \chi_I = \chi_{IIa} = \chi_{II} \\[4pt] \chi_{IIb} = \chi_{III} \end{array}$$

The criterion for instability including a density gradient is then given by

$$\eta \geq \left(1 - \frac{\chi_{III}}{\chi_{II}}\right),$$

again for the limit $Q_1 \ll Q_2$ and $a \simeq b$.

Now there is a strong stabilizing effect if $\chi_{III} < \chi_{II}$ and destabilization in the opposite case, and, of course, for constant density ($\eta = 0$) the criterion for stability is simply

$$\chi_{III} < \chi_{II}$$

The conclusion for a real plasma is that a heat condution coefficient which decreases towards the plasma centre, especially across the inner part of the radiating layer, effectively stabilizes the sheath. Fortunately, in experiment the heat conduction coefficient seems to exhibit just this behaviour. A region of ergodic field lines or a separatrix near the plasma boundary would also act in the right way.

If, however, there is an internal island close to the inner boundary of the radiating layer, then instability occurs which may destroy the sheath or, at best, result in a new stable equilibrium. This island effect could have some relevance for the disruption in an impure tokamak plasma / 8 /.

So, in addition to <u>equilibrium</u> control by $\chi(x)$ we find that the <u>stability is</u> also sensitive to $\chi(x)$, offering another "knob" for external control.

δ) Jump of χ in region I or III

The same procedure as before is applied to derive the dispersion relation and the stability criterion. It turns out, however, that the marginal point is not influenced and the stable regime is the same as with constant χ over the sheath, i.e. the stability criterion remains unchanged. Only the growth rates and eigenfunctions away from the marginal point are slightly affected. A χ variation outside the radiating layer is therefore not expected to influence the stability directly. Of course, there is an indirect influence since any change in a physical parameter changes the equilibrium, as discussed earlier.

e) Temperature dependent heat conduction coefficient $\chi(T)$

This case, though possibly of practical interest, is not treated quantitatively in this

paper since the mathematical procedure would be quite different, requiring a completely new analysis. It should also be noted that $\chi \propto T^n$ with $n < 0$ leads to an inherently unstable situation even in a pure heat conduction sheath. In order to get stable plasmas, one needs additional stabilizing mechanisms like those described above. The effect of additional radiation losses is not a priori obvious.

A numerical model / 9 / developed as an extension of the present one, indicates stability also in this case.

IV. Relevance of the Model for Non-Stationary Plasmas

The equilibrium and stability results are only applicable in a stationary or quasistationary phase of a discharge where the plasma parameters change on a time scale which is much longer than that of the eigenmodes of the radiating sheath. The latter is of the order of the heat condution time over the radiation layer and therefore indeed much smaller than the average energy confinement time, at least if heat conduction is dominant in the bulk plasma. The average particle confinement time is usually assumed to be of the order of the energy confinement time, too. The stationary state will usually be achieved during the flat-top phase of an experiment, e.g. during thermonuclear burning in ZEPHYR.

Major problems are to be expected during the heating and shut-down phase: Firstly, the loss power appreciably changes during that period and, in order to get all the power radiated away, the impurity content must be appropriately adjusted. Fortunately, there seem to be self-regulating mechanisms as indicated by the BALDUR results described, at least if the variation is slow enough. The linearized stability analysis is then still meaningful at any instant.

During switching of the heating power the problem is more complidated. On the one hand, the average loss power from the central plasma does not change to first order since the energy confinement time and the plasma energy content do not jump when the heating is switched on or off. The additional heating power is mainly converted into internal energy. But, looking at the plasma in more detail, one finds that appreciable local changes can occur. As an example, let us look at neutral injection. There is a substantial part of the beam

energy deposited in the plasma boundary. If the radiation sheath was in equilibrium before neutral injection, then it is suddenly transformed to a non-equilibrium. In order to describe the subsequent evolution, we need a one-dimensional, nonlinear and time-dependent transport model (such as BALDUR). The present sheath model can at best indicate how the sheath might develop by looking for neighbouring equilibria and their stability. Of course, a slow, continuous variation of the heating power would be desirable in order to retain quasi-stationary conditions.

A special ZEPHYR feature is the adiabatic major radius compression, which changes all plasma parameters within much less than an energy confinement time according to adiabatic scaling laws. Thus, an initial sheath equilibrium is transformed into a well-defined final state, which again will be a non-equilibrium and the same holds as before. If the deviation of the final state from a neighbouring equilibrium is not too large, then it may be treated as a small perturbation and the stability criterions are applicable. Further considerations on the non-stationary evolution of the radiating layer will be given elsewhere.

V. Summary and Conclusions

In one-dimensional plasma transport simulations with the BALDUR code including iron sputtering we have discovered a radiation cold plasma layer, radiating away nearly all the energy lost from the central plasma. Very fundamental arguments described elsewhere e.g. /4/ support the existence of such a layer in large fusion machines.

In order to understand the properties and the scaling of that boundary layer, a discontinuous sheath model is developed which yields simple analytic expressions for the equilibrium parameters and for stability criterions. Growth rates and eigenfunction are also easily derived.

The sheath model includes heat conduction, radiation and additional heating. The radiation characteristic is represented as a multistep function of temperature, resulting in spatial discontinuities. The heat conduction coefficient may jump in space, and radiation losses and additional heating may jump or change continuously. A density gradient is

allowed and temperature and heat flux change continuously in space anyway.

With respect to equilibrium the scaling with the heat conduction coefficient and with additional heating is most interesting since it offers the possibility of external control of the radiating layer. For instance, the total radiated power can be increased at constant impurity content by increasing χ , e.g. by ergodization of field lines. The equilibrium relations are also useful for comparing different types of impurities etc.

The stability against temperature disturbances or, equivalently against a spatial shift of the internal free boundaries (discontinuities) is investigated. Global stability criterions for a variety of equilibria are derived. For the most interesting case where most of the outgoing energy is radiated away, the following main tendencies are obtained:

- For spatially constant density and heat conduction, temperature independent additional heating and a single-step, box-type radiation characteristic the sheath is weakly stable irrespective of the geometrical dimensions and the radiated power fraction.

- A density gradient such that the product of electron times impurity density (determining the radiated power) increases in the same direction as the temperature, is destabilizing.

- The high-temperature wing in the radiation characteristic, causing radiation extending into the hot bulk plasma is stabilizing.

- A temperature dependent heat deposition or loss is stabilizing if the heating power decreases or the loss increases with temperature. (Important examples of such a mechanism are ohmic heating and ionization of inflowing neutrals). This agress with the qualitative condition that the total power loss should increase with temperature to get stability.

- Strong stabilization is obtained if the heat conduction χ decreases spatially towards the plasma centre, especially in the hotter part of the radiating layer. (Experiments indicate just this behaviour!)

Growth rates were calculated for the case with destabilizing density gradients. The result indicates that the analytic stability criterion is exact. The maximum growth rates are of

the order of the inverse heat conduction time over the radiation layer. The eigenfunction is also calculated for an unstable case.

The results of this analytic model may be useful to understand the dynamics of a radiating boundary layer in numerical plasma simulations and in experiments, and it may also serve as a useful guideline for them.

Figure Captions

Fig. 1a: Temperature and density profiles for a burning fusion plasma
(R = 135 cm, a = 50 cm, B = 9.14 T) calculated with the BALDUR code
(including iron sputtering). A cold plasma mantle ($T_e \leqslant 10$ eV) has developed.
The position of the radiation layer is indicated.

Fig. 1b: Heating and loss power within a cylinder of radius r versus r (same case as
Fig. 1a). The major loss mechanisms are electron heat conduction in the
centre and radiation in the plasma mantle. The α -heating exceeds the total
losses everywhere, i.e. the plasma is globally ignited.

Fig. 2: Power balance (as described in Fig. 1b) during RF heating. The computational
grid is refined in the edge region in order to get a better spatial resolution
of the radiation layer.

Fig. 3: The box-type radiation characteristics used in the model.

Fig. 4: Basic model sheath structure. The radiation power density P, the heat
flow Q and the temperature T are given as function of the distance X
from the limiter at X = 0.

Fig. 5: Sheath structure including space-dependent heating, H(x); the simple case
with piecewise constant heating is sketched.

Fig. 6: Sheath structure with spatially varying radiation power density P(x) caused
by changing electron- and impurity density, n_e and n_i.

Fig. 7: Sheath structure assuming a multistep radiation characteristic.

Fig. 8: The function D (ε) and its behaviour for $\varepsilon \rightarrow 0$ and $\varepsilon \rightarrow \infty$.
If D'' (0) $<$ 0 there is at least one positive ε where D (ε) $\equiv$ 0.

Fig. 9a: Normalised growth parameter $\varepsilon \cdot b$ versus $V = P_2/P_1$ for two different boundary conditions, showing only a small difference for large V.

Fig. 9b: Normalised growth paramter $\varepsilon \cdot b$ for small $V = P_2/P_1$. The analytic stability criterions are indicated also. They coincide with the numerial stability boundaries.

Fig. 10: Eigenfunction $\tau(x)$ for an unstable equilibrium with spatially varying impurity density and therefore varying radiation power density $P(x)$.

Figs. 11a, 11b: Sheath structure including a jump of χ at a point x^* near x_1 and x_2, respectively.

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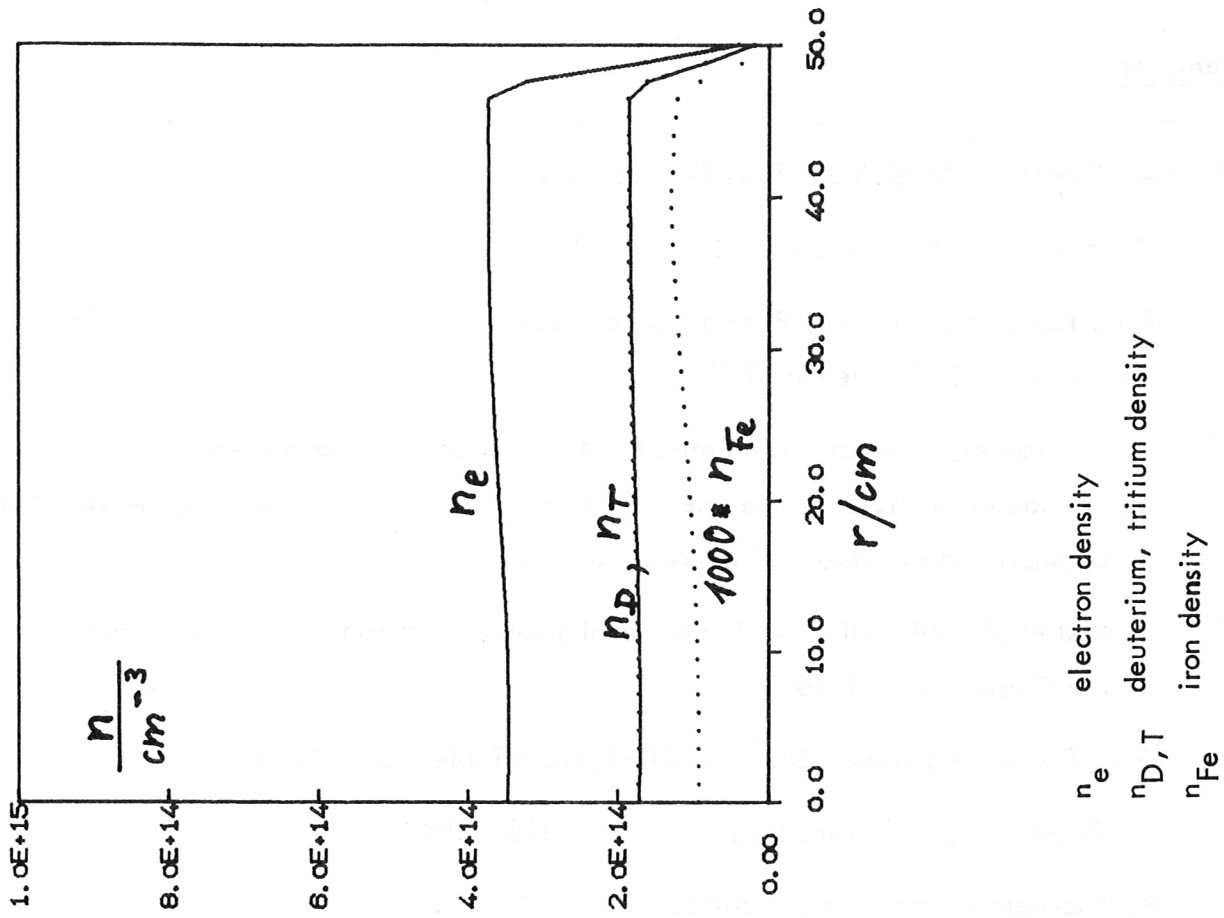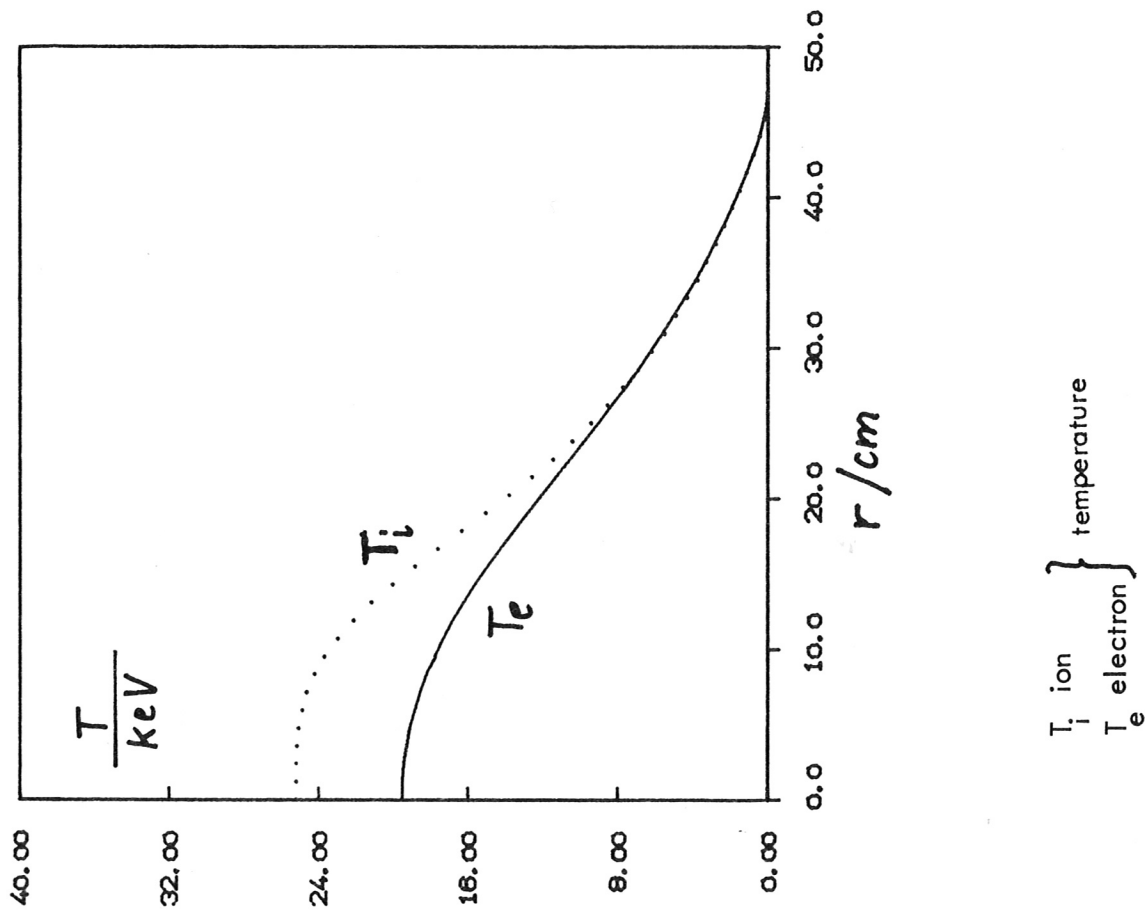
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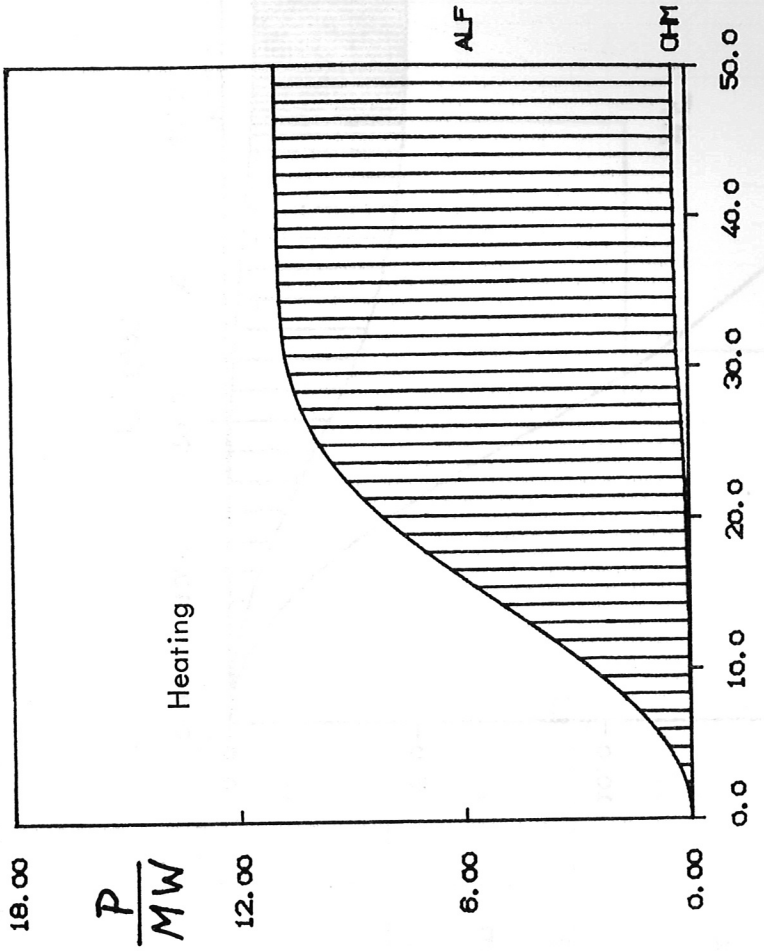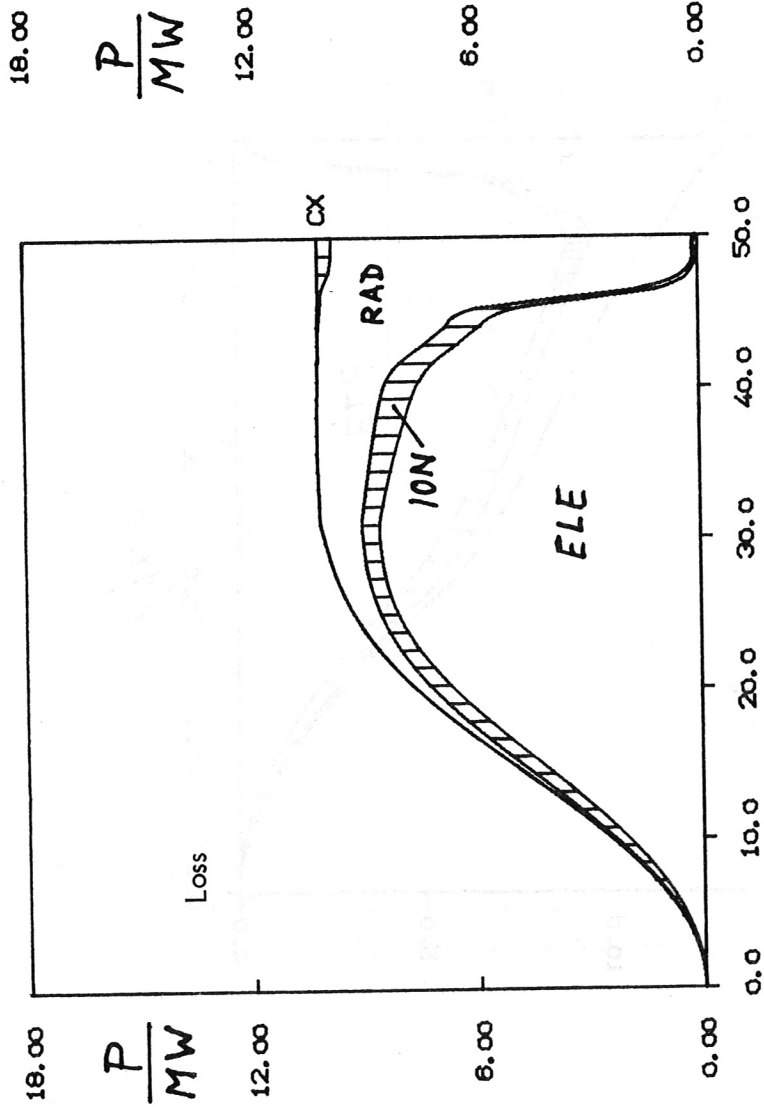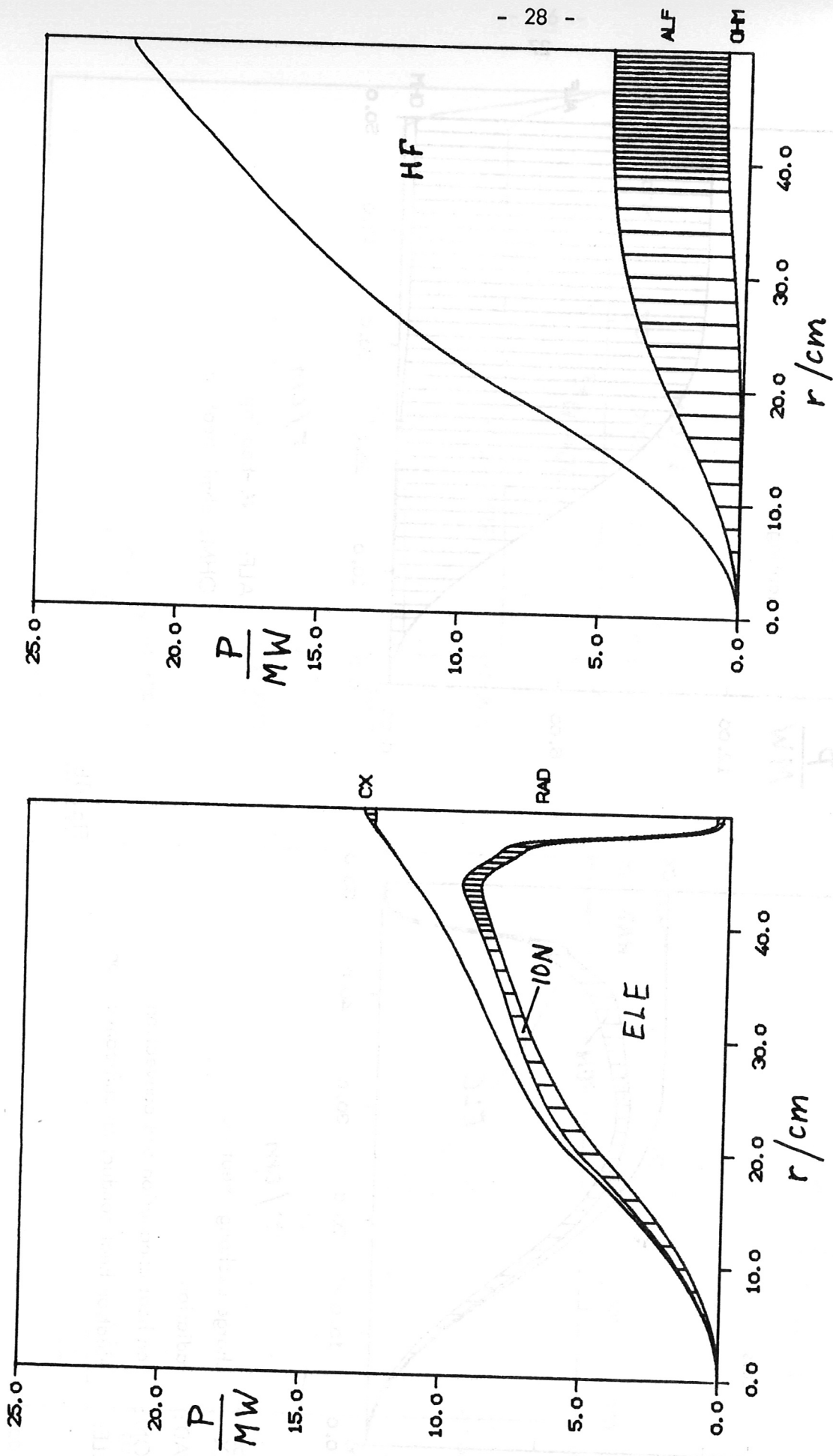
Fig. 1b

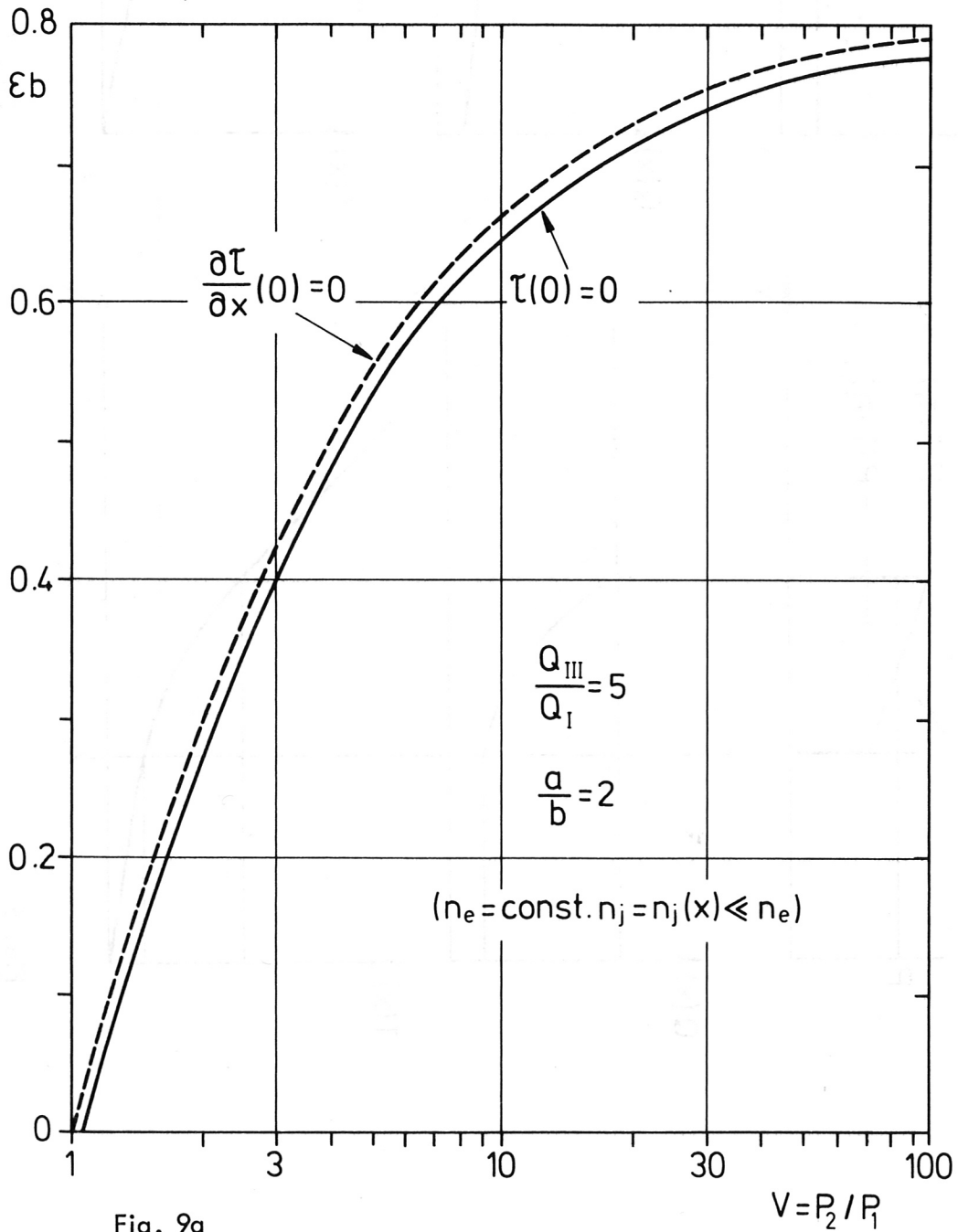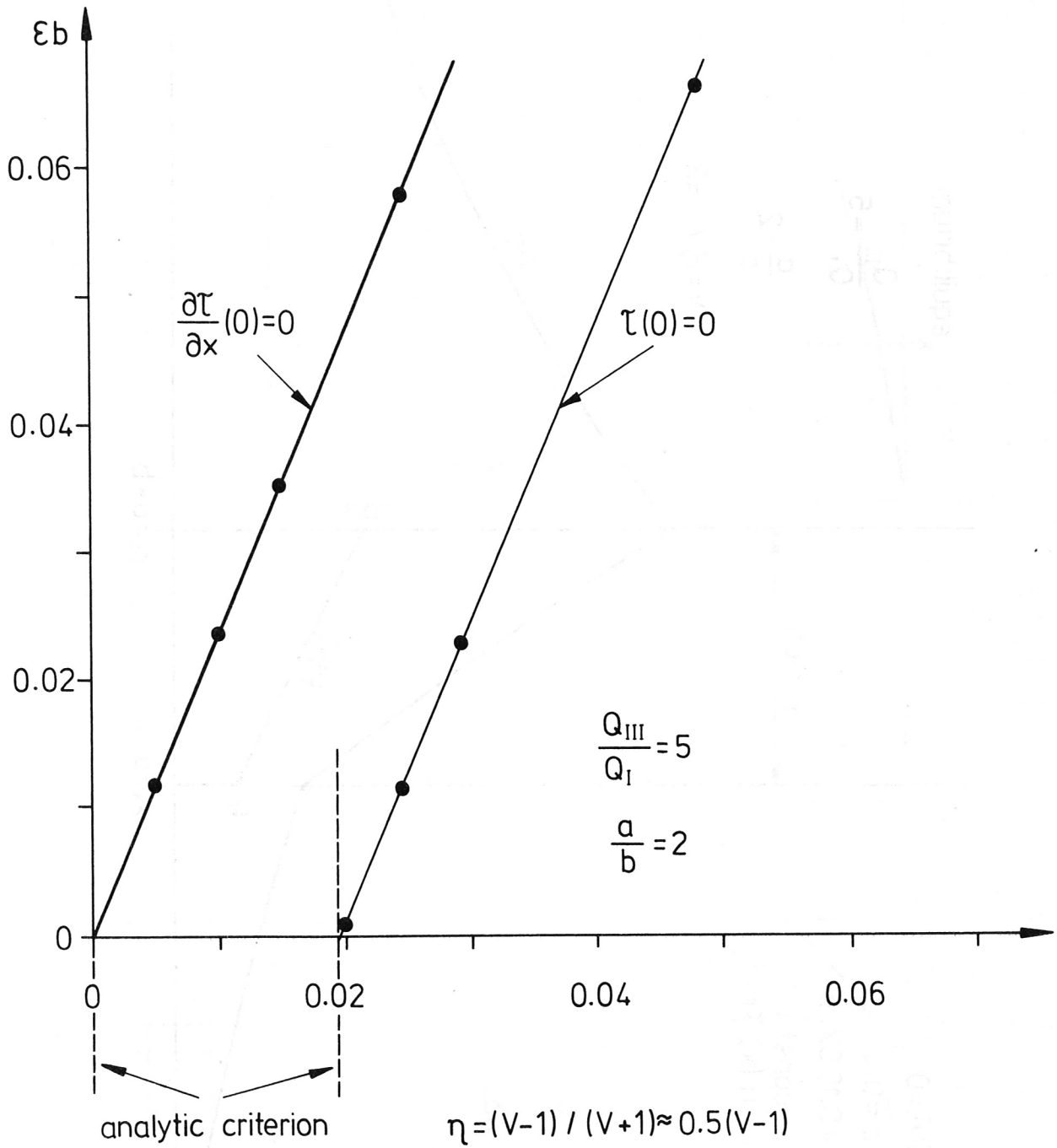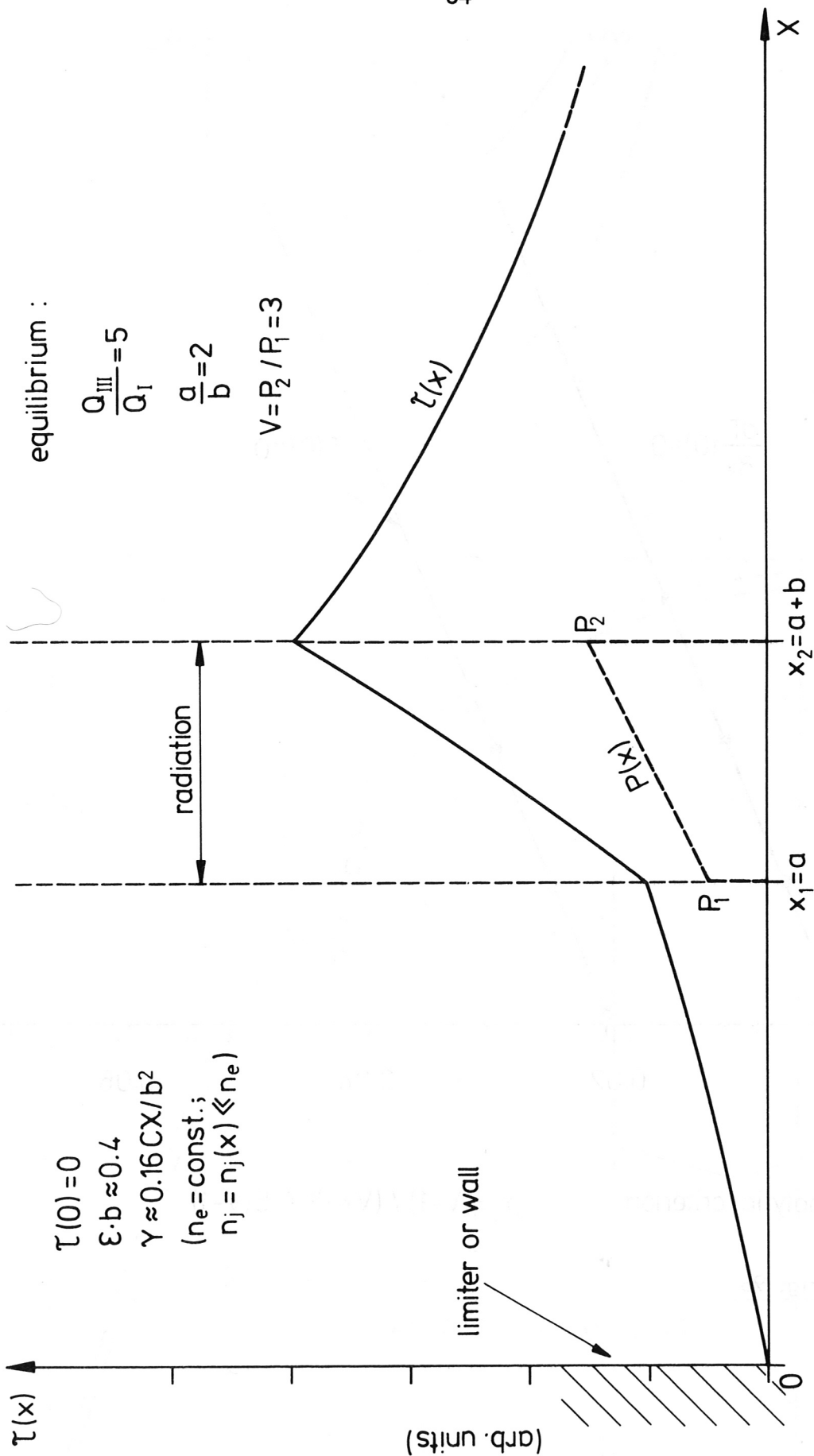HF: high-frequency heating; other abbreviations : see Fig. 1b.

Fig. 2
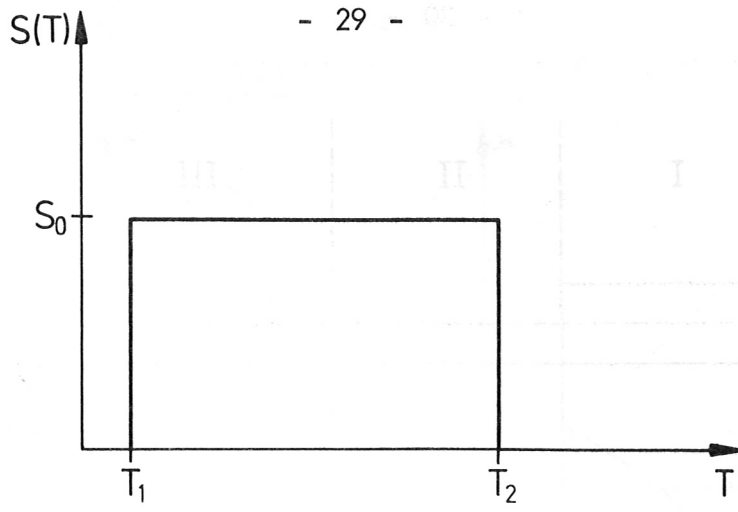
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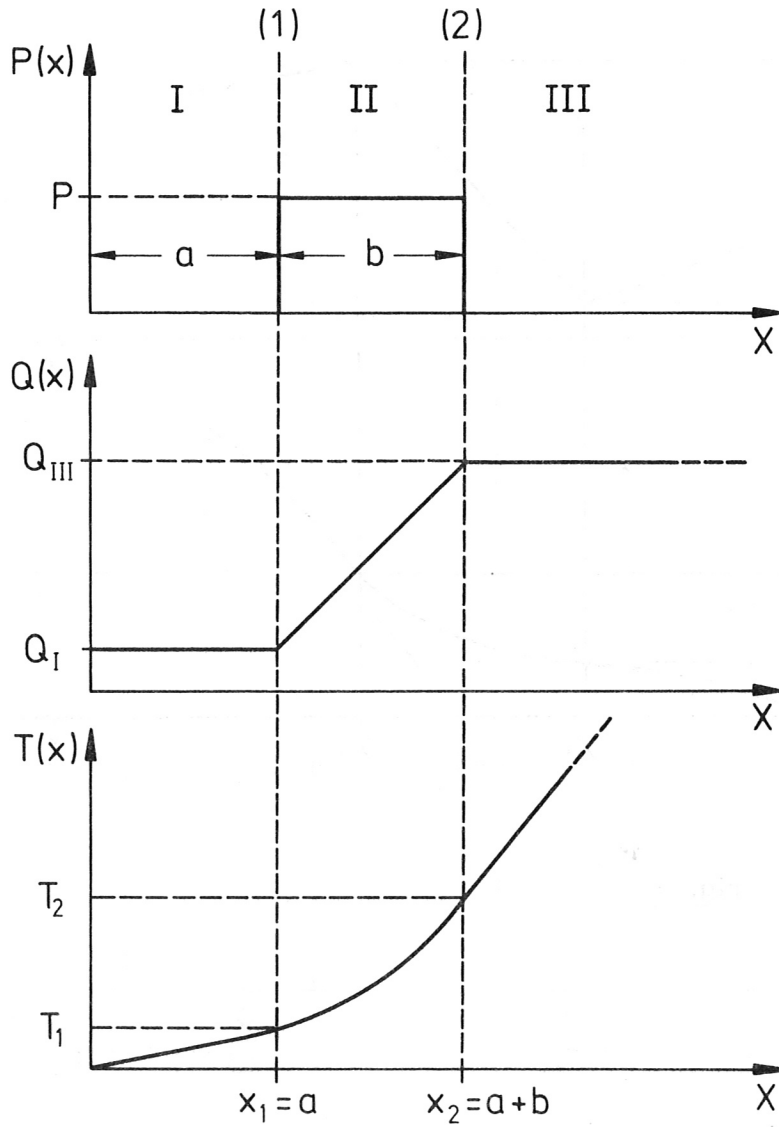
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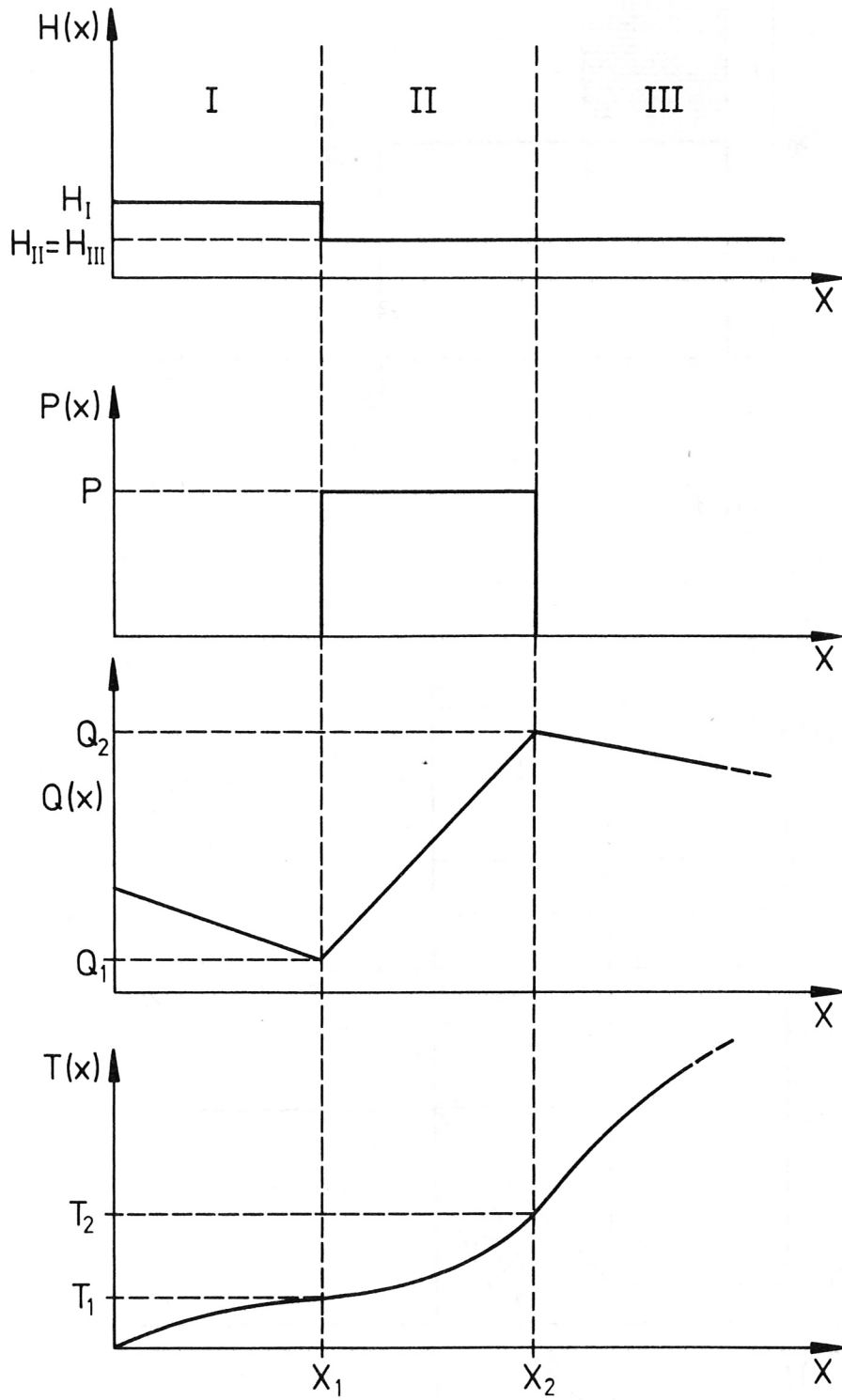
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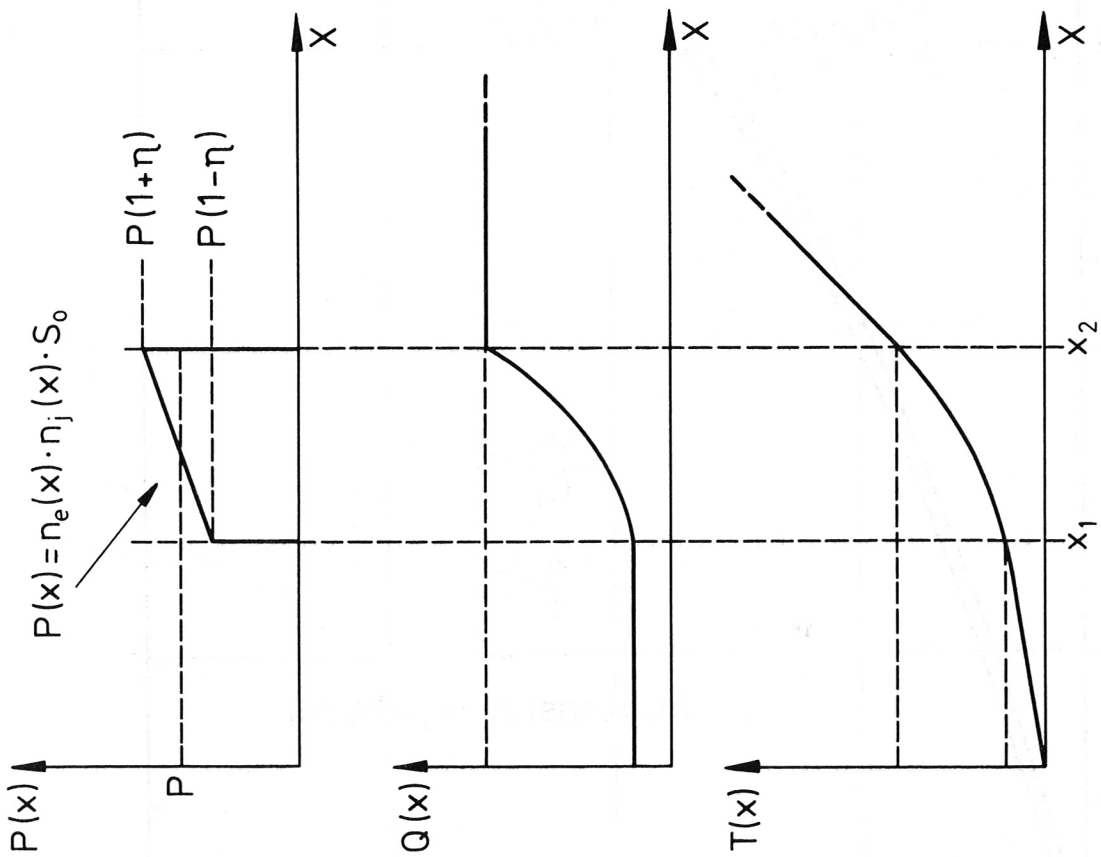
Fig. 7

Fig. 6
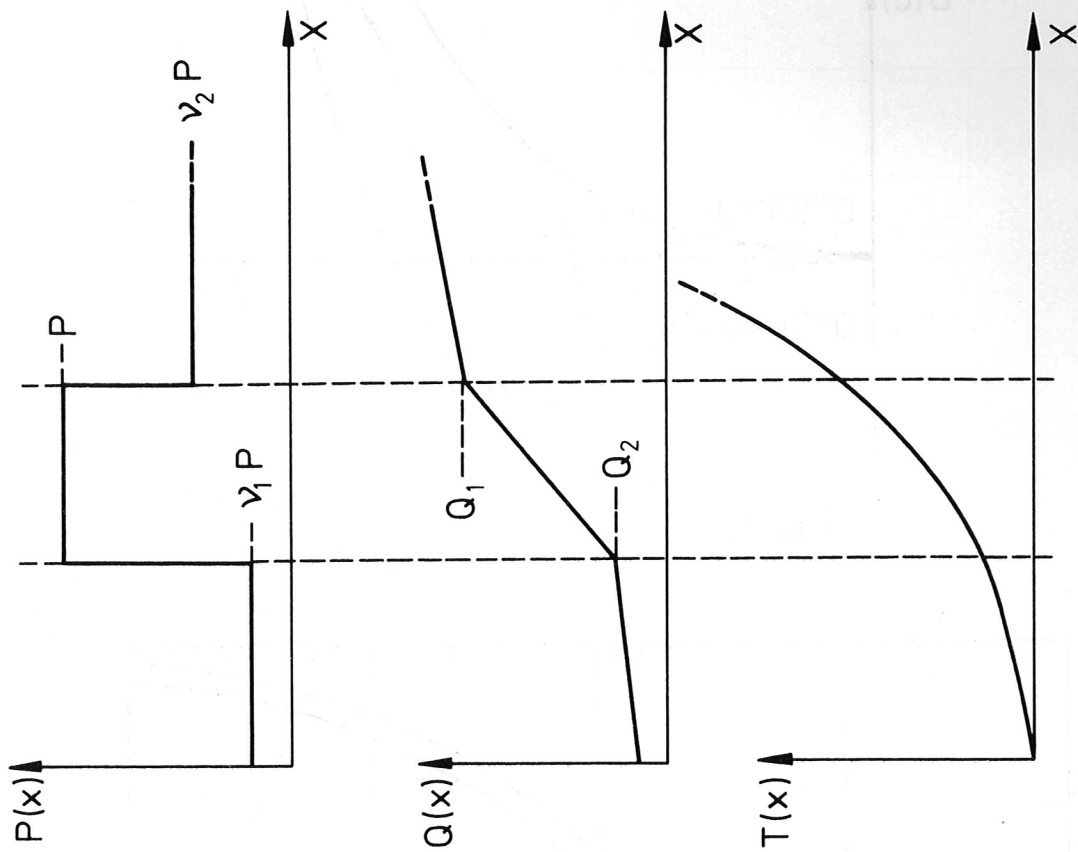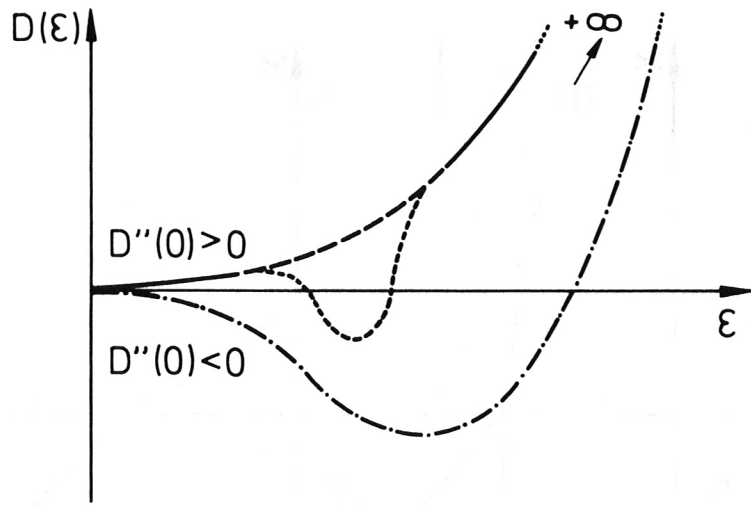
Fig. 8

Fig. 9a

Fig. 9b

Fig. 10

Fig. 11a

Fig. 11b